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EFFECT OF FINANCIAL CONSTRAINT ON THE TIME COST TRADE-OFF PROBLEMS USING RENEWABLE & NON- RENEWABLE RESOURCES

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ABSTRACT

The time cost trade-off problem is a type of project scheduling problem which studies how to transform project activities so as to achieve the trade-off between the project cost and the completion time. In the real projects, the trade-off between project cost and project completion time and the financial constraint with respect to renewable and non- renewable resources are considerable aspects of decision makers. In this paper, the impact of financial constraint on time cost trade-off problems using renewable and non-renewable resources is presented.

Key words: Time cost trade-off, renewable and non-renewable resources**,** financial constraint, AOA and AON

INTRODUCTION

An acyclic and directed project network that depicts precedence relationships between activities and includes unique start and finish nodes this is termed as activity on arc (AOA). Where as in the activity on node (AON) network each activity is represented by a node and each arc is the symbolic representation of precedence relationships between two activities. In conventional networks many times dummy activities have to be included to represents the precedence relationship between the activities to maintain the logic, this results in increase number of activities. As presented above all activities in AON network are represented by nodes or circles and all arrows in AON network are like dummies as they do not require any time or resources which represents only the relationships between the activities and moreover no dummy activities are required. The project is modeled by an activity on arc network (AOA), where its arcs represent project activities and its nodes define specific events. AON networks can be converted into AOA networks, i.e., activities on nodes are can be converted to activities on arcs where arcs suggest the flow of work or the progress of the project, while activities on nodes make the network appear static. The users of arc diagram argue that the identification of the activities in numeric form, that is by the tail and head event numbers (i,j) makes it more suitable for computer programming. Hence AON networks are converted into AOA network and procedure of AOA network are applied to AON network for solving the above said models. The objective is to construct the complete and efficient time, cost profile with respect to financial constraint of a project. Few mathematical models have been developed to minimize duration and cost by considering renewable, non-renewable resources, which relates the shortest project duration with respect to the budgetary constraint. The project has to be supported financially and all previous approaches presented earlier solved the time cost trade-off problems by assuming that the project budget as the fixed amount i.e., the project would never deviate from the original estimate which is non-renewable. Sometimes the assumptions are unrealistic as the project is generally funded by one or several external financing sources which includes financial institutions, investors and government, as well as via various financial alternatives such as loans, bonds etc. And these financial alternatives are connected with various levels of market, business and political risk. Therefore the project might be forced to stop either temporarily or even permanently, in either of the cases the project cannot be completed in acceptable time frame. Therefore it is important to compute and incorporate the financial constraint into the time cost trade – off problems.

PROBLEM FORMULATION

A project scheduling network is a set of precedence-constrained relations that has to be scheduled so as to minimize a given objective. In project scheduling network, the jobs additionally compete for limited resources. Due to its universality, the latter problem has a variety of applications in manufacturing, production planning, project management etc. It is one of the most difficult problems in operations research, and has therefore become an accepted platform for the latest optimization techniques, which includes virtually all local search paradigms. An activity is completed in number of ways, each of which corresponds to time and cost requirements. Since the acceleration of an activity requires additional resources and hence contributes to a higher cost. The main objective is to find the sets of decisions that lead to desirable project duration and cost. This process is called as the time cost trade-off analysis. In the time cost trade-off problems the decision maker has to decide to crash the activities to meet a required deadline or wants to evaluate whether crashing the activities are worth with the additional cost as presented in (1). The two objectives of time cost trade-off problems are to minimize cost and to minimize duration, this solution is termed as Pareto optimal i.e., there is no other shorter duration under a given budget or no lower cost under the required duration as presented in (2). The Pareto optimal is also termed as Minimum time cost curve, which is negatively sloped to the origin of time cost coordinate axis. Time cost trade-off problems have been developed from 1950's parallel with the development of critical path method. The objective is to construct the complete and efficient time, cost profile with respect to financial constraint of a project. Few mathematical models have been developed to minimize duration and cost by considering renewable, non-renewable resources, which relates the shortest project duration with respect to the budgetary constraint. Few linear programming models of AOA network to minimize the project duration subject to the financial constraint were presented.

NOTATIONS AND MATHEMATICAL MODELS

C_c = crash cost of an activity
N_c = normal cost of an activity
N_T = normal duration of an activity
C_T = crash duration of an activity
E_i = event time of a node i
T_{req} = project delivery time
S = start of an activity
f = finish of an activity
t_{ij} = the project duration of an activity i,j
$C_{ij}*$ = total cost of an activity i, j
C_i*= cost of an activity i
t_i = duration of an activity i
Es_i = earliest start time of an activity i
Ef_j = earliest finish time of an activity i
r_{ij} = the amount of renewable resource r
P_r = price of the renewable resource r
$C_{ij}**$ = cost of the activity (i,j) = $t_{ij} \times r_{ij} \times P_r$ + the cost of non renewable resources

The linear programming model of AOA network to minimize the project duration subject to the financial constraint

i) Minimize

$$E_j$$

subject to

$$\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij} t_{ij} \le B_{\max}$$

where $B_{\max}$ is maximum available budget

$$E_i + t_{ij} - E_j \le 0 \forall(i,j)$$

$$C_T \le t_{ij} \le N_T, 0\forall(i,j)$$, Boundary condition

$$E_s = 0,\ E_j \le T_{req}$$

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$$E_j, t_{ij} \geq 0$$

Where A_{ij}, B_{ij} and $C_{ij}*$ are cost slope, intercept on the y-axis (fig :) i.e., cost axis and cost of an activity respectively, are given by

$$A_{ij} = \frac{C_c - N_c}{N_T - C_T}$$

$$B_{ij} = C_c - A_{ij} C_T$$

$$C_{ij}* = B_{ij} + A_{ij} t_{ij}$$

If the study interchanges the objective function and constraints will be of the form

ii) Maximize

$$E_j$$

subject to

$$\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij} t_{ij} \leq B_{\max}$$

where $B_{\max}$ is maximum available budget

$$E_i + t_{ij} - E_j \leq 0 \forall(i, j)$$

$C_T \leq t_{ij} \leq N_T, 0\forall(i, j)$, Boundary condition

$$E_s = 0, \quad E_j \geq T_{req}$$

$$E_j, t_{ij} \geq 0$$

iii) Minimize

$$\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij} t_{ij}$$

Subject to

$$E_i + t_{ij} - E_j \leq 0 \forall(i, j)$$

$C_T \leq t_{ij} \leq N_T, 0\forall(i, j)$, Boundary condition

$$E_s = 0, \quad E_j \leq T_{req}$$

$$E_j, t_{ij} \geq 0$$

iv) Maximize

$$\sum_{\forall(i,j)} C_{ij}* = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij} t_{ij}$$

Subject to

$$E_i + t_{ij} - E_j \leq 0 \forall(i, j)$$

$C_T \leq t_{ij} \leq N_T, 0\forall(i, j)$, Boundary condition

$$E_s = 0, \quad E_j \geq T_{req}$$

$$E_j, t_{ij} \geq 0$$

The linear programming model of AON network to minimize the project duration subject to the budgetary constraint

v) Minimize

$$E_j$$

subject to

$$\sum_{\forall i} C_i * = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i \le B_{\max}$$

where $B_{\max}$ is maximum available budget

$$E_i + t_i - E_j \le 0 \forall i$$

$C_T \le t_i \le N_T, 0 \forall i$, Boundary condition

$$E_s = 0,\ E_f \le T_{req}$$

$$E_i, t_i \ge 0$$

Where A_i, B_i and C_i * are cost slope, intercept on the y-axis i.e., cost axis and cost of an activity respectively and $B_{\max}$ denotes the level of budget and is expressed as the random variable in lieu of the fixed estimate and the constraint is called the financial constraint.

vi) Maximize

$$E_j$$

subject to

$$\sum_{\forall i} C_i * = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i \le B_{\max}$$

where $B_{\max}$ is maximum available budget

$$E_i + t_i - E_j \le 0 \forall (i, j)$$

$C_T \le t_i \le N_T, 0 \forall i$, Boundary condition

$$E_s = 0,\ E_f \ge T_{req}$$

$$E_i, t_i \ge 0$$

vii) Minimize

$$\sum_{\forall i} C_i * = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i$$

Subject to

$$E_i + t_i - E_j \le 0 \forall (i, j)$$

$C_T \le t_i \le N_T, 0 \forall i$, Boundary condition

$$E_s = 0,\ E_f \le T_{req}$$

$$E_i, t_i \ge 0$$

viii) Maximize

$$\sum_{\forall i} C_i * = \sum_{\forall i} B_i + \sum_{\forall i} A_i t_i$$

Subject to

$$E_i + t_i - E_j \le 0 \forall (i, j)$$

$C_T \le t_i \le N_T, 0 \forall i$, Boundary condition

$$E_s = 0,\ \ E_f \geq T_{req}$$
$$E_i, t_i \geq 0$$

The linear programming model for AOA network using renewable and non renewable resources to minimize the project duration subject to the budgetary constraint

i) Minimize

$$E_j$$

subject to

$$\sum_{\forall(i,j)} C_{ij}{}^{**} = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij} \leq B_{\max}$$
$$E_i + t_{ij} - E_j \leq 0 \forall (i, j)$$
$$C_T \leq t_{ij} \leq N_T, 0 \forall (i, j)\text{, Boundary condition}$$
$$E_s = 0,\ \ E_j \leq T_{req}$$
$$E_j, t_{ij} \geq 0$$

Where $B_{\max}$ denotes the level of budget and is expressed as the random variable in lieu of the fixed estimate and the constraint is called the financial constraint.

ii) Maximize

$$E_j$$

subject to

$$\sum_{\forall(i,j)} C_{ij}{}^{**} = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij} \leq B_{\max}$$
$$E_i + t_{ij} - E_j \leq 0 \forall (i, j)$$
$$C_T \leq t_{ij} \leq N_T, 0 \forall (i, j)\text{, Boundary condition}$$
$$E_s = 0,\ \ E_j \geq T_{req}$$
$$E_j, t_{ij} \geq 0$$

iii) Minimize

$$\sum_{\forall(i,j)} C_{ij}{}^{**} = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij}$$

Subject to

$$E_i + t_{ij} - E_j \leq 0 \forall (i, j)$$
$$t_{ij} \leq N_T,\ \ t_{ij} \geq C_T$$
$$E_s = 0,\ \ E_j \leq T_{req}$$
$$E_j, t_{ij} \geq 0$$

Where $C_{ij}{}^{**}$ is the cost calculated using renewable and non renewable resources and is given by

$C_{ij}{}^{**}$ = cost of the activity (i,j) = $t_{ij} \times r_{ij} \times P_r$ + the cost of non renewable resources

α_{ij} is the cost of non-renewable resources

vi) Maximize

$$\sum_{\forall(i,j)} C_{ij}^{**} = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij}$$

Subject to

$$E_i + t_{ij} - E_j \leq 0 \forall (i,j)$$

$$t_{ij} \leq N_T, \quad t_{ij} \geq C_T$$

$$E_s = 0, \quad E_j \geq T_{req}$$

$$E_j, t_{ij} \geq 0$$

Where C_{ij}^{**} = cost of the activity (i,j) = $t_{ij} \times r_{ij} \times P_r$ + the cost of non renewable resources

SOLUTION PROCEDURE

For an AOA network, a LP model which includes objective function and constraints is formulated with respect to financial constraint without considering the renewable and non-renewable resources. And for the same AOA network, a LP model is formulated with financial constraint by considering renewable and non-renewable resources. Finally, these two models are illustrated by an example, and both the models are compared.

NUMERICAL EXAMPLE

The data of Housing construction by SSR Constructions is summarized in Table-I in which there are 11 activities with maximum available budget ₹.78,500, with shortest possible duration of 17 weeks to complete the project at least cost within maximum available budget and desired project completion time is 18 weeks

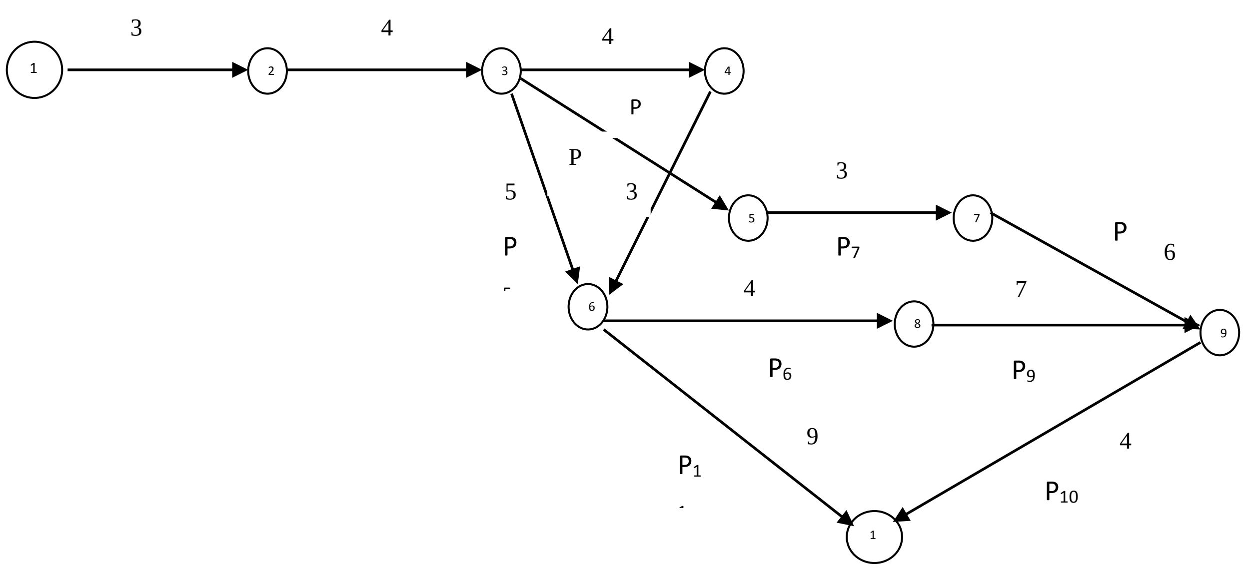

Fig I: Project Network of House Construction project

Table 1: Housing Project description

Activity	Activity Name	Predecessor	Normal Time	Crash Time	Normal Cost	Crash Cost	Max Reduction in time	Cost Slope (A_{ij})
1-2	P_1	---	3	2	5000	7000	1	2000
2-3	P_2	P_1	4	2	4000	5000	2	500
3-4	P_3	P_2	4	4	7000	7000	0	---
3-5	P_4	P_2	3	1	3000	5000	2	1000
3-6	P_5	P_2	5	2	6000	10500	3	1500
6-8	P_6	P_5, P_3	4	3	8000	10000	1	2000
5-7	P_7	P_4	3	1	4000	5500	2	750
7-9	P_8	P_7	6	4	6000	9000	2	1500
8-9	P_9	P_6	7	4	5000	8000	3	1000
9-10	P_{10}	P_8, P_9	4	2	6000	7500	2	750
6-10	P_{11}	P_3, P_5	9	7	3000	4000	2	500

Cost intercepts for the project are calculated using the equation $B_{ij} = C_c - A_{ij} C_T$

For example for activities 1-2 and 2-3

B_{12} = 7000 – 2000 × 2 = 3000

B_{23} = 5000 – 500 × 2 = 4000 etc.

Now substituting these values in the model (i), we get
Minimize

E_{P11}

$= E_{610}$

Subject to constraints
First constraint

$$\sum_{\forall(i,j)} C_{ij} * = \sum_{\forall(i,j)} B_{ij} + \sum_{\forall(i,j)} A_{ij} t_{ij} \le B_{\max}$$

$$= C_{P1} + C_{P2} + C_{P3} + C_{P4} + C_{P5} + C_{P6} + C_{P7} + C_{P8} + C_{P9} + C_{P10} + C_{P11}$$

$$= \left(B_{P1} + B_{P2} + B_{P3} + B_{P4} + B_{P5} + B_{P6} + B_{P7} + B_{P8} + B_{P9} + B_{P10} + B_{P11}\right) +$$

$$\left(2000t_{P1} + 500t_{P2} + 0t_{P3} + 1000t_{P4} + 1500t_{P5} + 2000t_{P6} + 750t_{P7} + 1500t_{P8} + 1000t_{P9} + 750t_{P10} + 500t_{P11}\right)$$

$$\leq B_{\max}$$

$$= (47750) + \left(2000t_{12} + 500t_{23} + 0t_{34} + 1000t_{35} + 1500t_{36} + 2000t_{68} + 750t_{57} + 1500t_{79} + 1000t_{89} + 750t_{910} + 500t_{610}\right)$$

$$\leq 78{,}500$$

$$= \left(2000t_{12} + 500t_{23} + 0t_{34} + 1000t_{35} + 1500t_{36} + 2000t_{68} + 750t_{57} + 1500t_{79} + 1000t_{89} + 750t_{910} + 500t_{610}\right)$$

$$\leq 30{,}750$$

Second set of constraint are given as follows, $E_i + t_{ij} - E_j \leq 0 \forall (i, j)$

For an activity 1-2 and 2-3 are

$E_1 + t_{12} - E_2 \leq 0$ $\quad$ $E_2 + t_{23} - E_3 \leq 0$

$i.e., E_1 + t_{12} \leq E_2$ $\quad$ $i.e., E_2 + t_{23} \leq E_3$

Third set of constraints are as follows, $C_T \leq t_{ij} \leq N_T, 0 \forall (i, j)$, Boundary condition

$2 \leq t_{12} \leq 3$, $2 \leq t_{23} \leq 4$, $t_{34} = 4$

$1 \leq t_{35} \leq 3$, $2 \leq t_{36} \leq 5$, $3 \leq t_{68} \leq 4$

$1 \leq t_{57} \leq 3$, $4 \leq t_{79} \leq 6$, $4 \leq t_{89} \leq 7$

$2 \leq t_{910} \leq 4$, $7 \leq t_{610} \leq 9$

$E_4 \leq E_6$, for the dummy activity 4 – 6

Fourth and fifth constraints are

$E_s = 0$, $E_j \leq T_{req}$, $E_1 = 0$, $E_{10} \leq 17$

Finally the non-negative constraint is

$$E_j, t_{ij} \geq 0$$

$$E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10} \geq 0$$

$$t_{12}, t_{23}, t_{34}, t_{35}, t_{36}, t_{68}, t_{57}, t_{79}, t_{89}, t_{910}, t_{610} \geq 0$$

The LP model is solved by running TORUS software and the proposed model can help to quantify the importance of budget and can help us to evaluate the financial risks in project management. Therefore with respect to the maximum available budget of Rs. 78,500, the minimum length of the project is 17 weeks. Now considering the same example to illustrate the LP models with respect to renewable and non- renewable resources

Minimize

$$E_j = E_{10}$$

Subject to

$$\sum_{\forall(i,j)} C_{ij} ** = r_{ij} P_r \sum_{\forall(i,j)} t_{ij} + \sum_{\forall(i,j)} \alpha_{ij} \le B_{\max}$$

$$= C_{P1} + C_{P2} + C_{P3} + C_{P4} + C_{P5} + C_{P6} + C_{P7} + C_{P8} + C_{P9} + C_{P10} + C_{P11}$$

$$= 4 \times 6 (t_{P1} + t_{P2} + t_{P3} + t_{P4} + t_{P5} + t_{P6} + t_{P7} + t_{P8} + t_{P9} + t_{P10} + t_{P11}) +$$

$$(\alpha_{P1} + \alpha_{P2} + \alpha_{P3} + \alpha_{P4} + \alpha_{P5} + \alpha_{P6} + \alpha_{P7} + \alpha_{P8} + \alpha_{P9} + \alpha_{P10} + \alpha_{P11})$$

$$\le B_{\max}$$

$$= (t_{12} + t_{23} + t_{34} + t_{35} + t_{36} + t_{68} + t_{57} + t_{79} + t_{89} + t_{910} + t_{610}) \le 896$$

Rest of the constraints are as the above model

The LP model is solved by running TORUS software and the proposed model can help to quantify the importance of budgets by using renewable and non-renewable resources. Therefore with respect to the financial constraint, renewable and non- renewable resources and with maximum available budget of Rs. 78,500, the minimum length of the project is 17 weeks. Similarly the rest of the models can be explained as above illustration.

CONCLUSION

In this paper, the two Linear programming models with impact of financial constraint one considering renewable and non-renewable resources and the other model without considering the renewable and non-renewable resources are developed. There is no much difference between the two models and the enumeration arithmetic to fit the cost of the project within the maximum available budget is adopted.

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